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Nonuniform Prices with Income Effects

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Abstract

The paper examines a reverse of the revelation principle when consumers' preferences have income effects. I propose a new method for constructing a price schedule from a direct revelation mechanism. Any incentive compatible direct revelation mechanism can be implementable by a single price schedule.

Keywords: nonuniform prices · income effects · taxation principle · bunching

JEL Classification Numbers: D4 · D4 · D11 · D82 · D86

1 Introduction

The paper studies the derivation of the price schedule as an indirect mechanism transformed from a direct revelation mechanism in a context with income effects. In the literature on price discrimination, such as Mussa and Rosen (1978) and Maskin and Riley (1984), mainly assumes no income effects. The usual motivation for ignoring income effects is that each consumer is not subject to a budget constraint in the sense that he spends a small fraction relative to his total expenditure on the good in question. However, income effects are not negligeble if differences in income across consumers may account for consumer heterogeneity. In the paper, consumers are endowed with heterogeneous income, and consumers' preferences are represented as non-seperable utility functions with income effects.

Without specifying the principal's objective, I will focus on consumer's utility maximization problem in a principal-agent problem under asymmetric information. There is adverse selection because the agent's characteristics/types are not observable to the principal (the distribution being

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known, however). The principal and the agent contract on a product characteristic such as quality and quantity, and a monetary transfer. The agent's product choice can be described as a *decision rule* that assigns a product choice for each type. The strategy space of the principal is a set of nonlinear price schedules. The standard approach to the screening problem is reformulating the principal's strategy space, due to the *revelation principle*.

The properties of contracts indexed by agent types, such as distortions and information rents, have been analyzed in the optimal contracts literature. In contrast, the purpose of the paper is to establish the reverse of the revelation principle in order to get back to a nonlinear price schedule as the principal's original strategy in a general setting in which the reservation utility is type-specific.

My concern in Section 4 is how to construct such a price schedule. Theorem 2 in the paper establishes the implementability of any decision rules possibly involving bunching when consumers have non-seperable utility functions with income effects. I introduce the notion of *voluntary implementability*, taking into account the agent's type-dependent reservation utility. Theorem 2, together with the revelation principle, states that any pair of a decision rule and an information rent is incentive compatible and individually rational if and only if it is voluntarily implementable by some indirect mechanism.

2 Utility Maximization Problem under Nonuniform Prices

There is a continuum of consumers in the economy. There are two commodities; a good x subject to nonuniform pricing $t(x)$, and a composite good y. Consumers have different levels of income. The distribution of income θ is defined on an interval Θ. A consumer with income θ maximizes $u(x,y)$ with $\nabla u(x,y) \gg 0$ subject to the budget constraint $y + t(x) \leqslant \theta$. Therefore, income effects are incorporated so that $u(x, \theta - t(x))$, beyond quasi-linearity.

Roberts (1979) adopts such a formulation of utility function parameterized by income.[1] The reduced form $u(x, \theta - t(x))$ of utility function boils down to consumers' preferences in vertical differentiation models. If the utility function is additively seperable in the consumption x and the residual income $y = \theta - t(x)$, that is, $u(x, \theta - t(x)) = v(x) + w(\theta - t(x))$, then the quasi-linear utility function $\tilde{\theta} v(x) - t(x)$, where $\tilde{\theta} = 1/w'(\theta)$ is the inverse of the marginal utility of income, is obtained as a linear approximation of the original utiilty function.[2] In this case, the parameter $\tilde{\theta}$ is interpreted as a *taste* in a vertical differentiation model. Therefore, the formulation of utility function $u(x, \theta - t(x))$ is not restricted to the analysis.

Roberts (1979) introduces the following regularity condition on preferences.

Assumption 1 (Normality)**.** The commodity x under a non-linear pricing scheme is *normal* in the sense that

$$N(x,y) = u_{xy}(x,y) - \frac{u_{yy}(x,y)}{u_y(x,y)} u_x(x,y) > 0.$$

[1] Subsequent works also assume the same formulation, such as Goldman et al. (1984, pp.315-316), Kanbur et al. (2000), and Wilson (1993, Chapter 7).

[2] See, Tirole (1988, pp.143-144).

Any additively seperable utility function $u(x,y) = v(x) + w(y)$ for some concave function $w : \mathbb{R} \to \mathbb{R}$ satisfies the normality condition. As long as the marginal utility $u_y(x,y)$ of residual income is positive, this assumption is satisfied if and only if the mariginal rate of substitution between x and y with respect to income θ is increasing. The monotonicity of the marginal rate of substitution has been discussed in the context of comparative statics.

$$\frac{\partial}{\partial\theta} MRS_{xy}(x,y) = \frac{\partial}{\partial\theta} \frac{u_x(x,y)}{u_y(x,y)} = \frac{u_{xy}(x,y)u_y(x,y) - u_x(x,y)u_{yy}(x,y)}{[u_y(x,y)]^2} = \frac{N(x,y)}{u_y(x,y)}.$$

Therefore, $\partial MRS_{xy}(x,y)/\partial\theta > 0$ if and only if $N(x,y) > 0$.

Finally, denote the product line by $X = [x(\underline{\theta}), x(\overline{\theta})]$, given a non-decreasing decision rule $x(\theta)$. The purpose of the present paper is to construct a price schedule under which any non-decreasing decision rule $x : \Theta \to X$ emerges as a solution to utility maximization problem of consumers.

3 Two Concepts of Implementability

According to Rochet (1985, 1987), there are two notions of implementability of decision rules. The literature on contract theory has been focused on the following incentive compatibility through a transfer function defined over the type space.

Definition 1. A decision rule $x(\cdot)$ is said to be *rationalizable or implementable via transfer* if there exists a *transfer function* $p : \Theta \to \mathbb{R}$ such that the direct revelation mechanism $\langle x(\cdot), p(\cdot)\rangle$ induces truthful revelation: $\theta \in \text{argmax}[u(x(\hat{\theta}), \theta - p(\hat{\theta})) \mid \hat{\theta} \in \Theta]$ for every $\theta \in \Theta$.

On the other hand, my concern is the implementability in the following sense.

Definition 2. A decision rule $x(\cdot)$ is said to be *implementable via price schedule* if there exists a price schedule $t : X \to \mathbb{R}$ such that $x(\theta) \in \text{argmax}[u(x, \theta - t(x)) \mid x \in X]$ for every $\theta \in \Theta$.

Rochet (1985, p.127) examines the reverse of the revelation principle, called the *taxation principle*. For a given rationalizable decision rule $x(\cdot)$ by a transfer function $p(\cdot)$, he proposes the price schedule $t : X \to \mathbb{R}$ defined as $t(x) = p$ if there exist θ such that $x(\theta) = x$ and $p(\theta) = p$ for each $x \in$ Range $x(\cdot)$. It is not difficult to see that $x(\theta)$ is a solution of the problem of maximizing $u(x, \theta - t(x))$ indeed.

In the paper, the participation constraints are incorporated into the implementability in the following manner. The agent of type θ may have an outside opportunity, form which he can derive a utility level $\bar{\pi}(\theta)$.

Definition 3. A direct revelation mechanism $\langle x(\cdot), p(\cdot)\rangle$ is *voluntarily implementable via price schedule* if there exists a price schedule $t : X \to \mathbb{R}$ such that for every $\theta \in \Theta$,
(1) $x(\theta) \in \text{argmax}[u(x, \theta - t(x)) \mid x \in X]$,
(2) $\max\,[u(x, \theta - t(x)) \mid x \in X] \geqslant \bar{\pi}(\theta)$.

A corresponding requirement for a direct revelation mechanism is a combination of incentive compatibility and individual rationality.

Definition 4. A direct revelation mechanism $\langle x(\cdot), p(\cdot)\rangle$ is *incentive compatible and individually rational* if for every $\theta \in \Theta$,
(1) $\theta \in \operatorname{argmax}[u(x(\hat{\theta}), \theta - p(\hat{\theta})) \mid \hat{\theta} \in \Theta]$,
(2) $u(x(\theta), \theta - p(\theta)) \geqslant \bar{\pi}(\theta)$.

The following observation is known as the *the revelation principle*. The proof is omitted.

Theorem 1 (Revelation Principle)**.** *If a direct revelation mechanism $\langle x(\cdot), p(\cdot)\rangle$ is voluntarily implementable, then it is incentive compatible and individually rational.*

My question is whether it is possible to construct a nonlinear price schedule for *any* given incentive compatible and individually rational direct revelation mechanism for the voluntary implementability. This paper proposes a procedure to construct a price schedule in order to incorporate information about type-dependent reservation utility.

4 Main Result

This section will be devoted to obtain the price schedule from any feasble direct mechanism. Let $\langle x(\cdot), p(\cdot)\rangle$ be incentive compatible direct revelation mechanism consisting of a decision rule and a transfer function which comes from some price schedule $T : X \to \mathbb{R}$ in the sense that $T(x(\theta)) = p(\theta)$. In this setting, the reservation utility of the agent with income θ is given by $\bar{\pi}(\theta) = u(0, \theta - T(0))$.[3] The corresponding information rent becomes

$$r(\theta) = u(x(\theta), \theta - p(\theta)) - \bar{\pi}(\theta).$$

The argument below starts with the principal who has found an optimal direct revelation mechanism $\langle x(\cdot), p(\cdot)\rangle$ or $\langle x(\cdot), r(\cdot)\rangle$.

Lemma 1. *If $\langle x(\cdot), p(\cdot)\rangle$ is incentive compatible, then $x(\cdot)$ is non-decreasing and the envelope condition $\dot{r}(\theta) = u_y(x(\theta, \theta - p(\theta))) - \dot{\bar{\pi}}(\theta)$ holds.*

Proof. The proof of the monotonicity of the decision rule can be found in Roberts (1979, p.82). I shall show the envelope condition. Let the indirect utility function be $U(\theta) = u(x(\theta), \theta - p(\theta))$. Since $U(\theta) \geqslant u(x(\hat{\theta}), \theta - p(\hat{\theta}))$, it follows that

$$u(x(\hat{\theta}), \hat{\theta} - p(\hat{\theta})) - u(x(\hat{\theta}), \theta - p(\hat{\theta})) = U(\hat{\theta}) - u(x(\hat{\theta}), \theta - p(\hat{\theta})) \geqslant U(\hat{\theta}) - U(\theta).$$

Similarly, since $U(\hat{\theta}) \geqslant u(x(\theta), \hat{\theta} - p(\theta))$, it follows that

$$u(x(\theta), \theta - p(\theta)) - u(x(\theta), \hat{\theta} - p(\theta)) = U(\theta) - u(x(\theta), \hat{\theta} - p(\theta)) \geqslant U(\theta) - U(\hat{\theta}).$$

[3] Roberts (1979) assumes that $T(0) = 0$.

Combining these inequalities to get

$$u(x(\hat{\theta}),\hat{\theta}-p(\hat{\theta}))-u(x(\hat{\theta}),\theta-p(\hat{\theta}))\geqslant U(\hat{\theta})-U(\theta)\geqslant u(x(\theta),\hat{\theta}-p(\theta))-u(x(\theta),\theta-p(\theta)).$$

Taking the limit as $\hat{\theta}\to\theta$, it must be the case that $\dot{U}(\theta)=u_y(x(\theta),\theta-p(\theta))$. In addition, by contraction, $U(\theta)=r(\theta)+\bar{\pi}(\theta)$ so that $\dot{r}(\theta)+\dot{\bar{\pi}}(\theta)=u_y(x(\theta,\theta-p(\theta)))$. This establishes the lemma. □

I will introduce an auxiliary function to define a particular price schedule for the voluntary implementation. By the strict monotonicity of $u(x,y)$ with respect to y, there is a unique $z=\phi(x,\hat{\theta})\in\mathbb{R}$ such that

$$r(\hat{\theta})=u(x,\hat{\theta}-z)-\bar{\pi}(\hat{\theta})$$

for each $x\in X$ and $\hat{\theta}\in\Theta$. For each $x\in X$, define a price schedule by

$$t(x)=\max[\phi(x,\hat{\theta})\mid\hat{\theta}\in\Theta].$$

The following lemma summarizes the properties of the funtion $\phi(x,\hat{\theta})$.

Lemma 2. *Let $\langle x(\cdot),p(\cdot)\rangle$ be any direct revelation mechanism. Then,*
(1) $\phi(x(\theta),\theta)=p(\theta)$ for every $\theta\in\Theta$.
(2) $\phi_x(x,\hat{\theta})=MRS_{xy}(x,\hat{\theta}-\phi(x,\hat{\theta}))$ for every $x\in X$ and every $\hat{\theta}\in\Theta$.
In addition, if $\langle x(\cdot),p(\cdot)\rangle$ is incentive compatible and individually rational then
(3) $\theta\in\text{argmax}[\phi(x(\theta),\hat{\theta})\mid\hat{\theta}\in\Theta]$ for every $\theta\in\Theta$.

Proof. The first assertion is immediate from the construction of $\phi(x,\hat{\theta})$. To obtain the expression for $\phi_x(x,\hat{\theta})$, totally differentiating the information rent $r(\hat{\theta})=u(x,\hat{\theta}-\phi(x,\hat{\theta}))-\bar{\pi}(\theta)$ with respect to x yields that

$$0=u_x(x,\hat{\theta}-\phi(x,\hat{\theta}))+u_y(x,\hat{\theta}-\phi(x,\hat{\theta}))\times[-\phi_x(x,\hat{\theta})],$$

and solving for $\phi_x(x,\hat{\theta})$:

$$\phi_x(x,\hat{\theta})=\frac{u_x(x,\hat{\theta}-\phi(x,\hat{\theta}))}{u_y(x,\hat{\theta}-\phi(x,\hat{\theta}))}=MRS_{xy}(x,\hat{\theta}-\phi(x,\hat{\theta})).$$

This establishes the second assertion.

To show the third assertion, suppose that the direct revelation mechanism $\langle x(\cdot),p(\cdot)\rangle$ is incentive compatible and individually rational. In order to verify the last assertion, it suffices to show that (a) $\phi_{\hat{\theta}}(x(\theta),\theta)=0$ and (b) the function $\phi(x,\hat{\theta})$ is concave in $\hat{\theta}$ at $x=x(\theta)$. To derive the expression for $\phi_{\hat{\theta}}(x,\hat{\theta})$, totally differentiating the equation that determines the function $\phi(x,\hat{\theta})$ with respect to $\hat{\theta}$ to obtain

$$\dot{r}(\hat{\theta})=u_y(x,\hat{\theta}-\phi(x,\hat{\theta}))\times[1-\phi_{\hat{\theta}}(x,\hat{\theta})]-\dot{\bar{\pi}}(\hat{\theta}).$$

On the other hand, the envelope condition due to the incentive compatibility is given as

$$\dot{r}(\hat{\theta}) = u_y(x(\hat{\theta}), \hat{\theta} - p(\hat{\theta})) - \dot{\pi}(\hat{\theta}).$$

Because of the fact that $p(\hat{\theta}) = \phi(x(\hat{\theta}), \hat{\theta})$,

$$u_y(x, \hat{\theta} - \phi(x, \hat{\theta})) \times [1 - \phi_{\hat{\theta}}(x, \hat{\theta})] = u_y(x(\hat{\theta}), \hat{\theta} - \phi(x(\hat{\theta}), \hat{\theta})).$$

Solving for $\phi_{\hat{\theta}}(x, \hat{\theta})$ to obtain the following:

$$\phi_{\hat{\theta}}(x, \hat{\theta}) = 1 - \frac{u_y(x(\hat{\theta}), \hat{\theta} - \phi(x(\hat{\theta}), \hat{\theta}))}{u_y(x, \hat{\theta} - \phi(x, \hat{\theta}))} = \frac{u_y(x, \hat{\theta} - \phi(x, \hat{\theta})) - u_y(x(\hat{\theta}), \hat{\theta} - \phi(x(\hat{\theta}), \hat{\theta}))}{u_y(x, \hat{\theta} - \phi(x, \hat{\theta}))}.$$

Therefore, the numerator becomes zero when $\hat{\theta} = \theta$ and $x = x(\theta)$. This implies that $\phi_{\hat{\theta}}(x(\theta), \theta) = 0$ for every $\theta \in \Theta$.

It remains to show that the second-order condition, $\phi_{\hat{\theta}\hat{\theta}}(x, \hat{\theta}) \leqslant 0$, is satisfied when $x = x(\hat{\theta})$. The second-order condition is written as

$$\phi_{\hat{\theta}\hat{\theta}}(x, \hat{\theta}) = \frac{\{(\text{Term 1}) - (\text{Term 2})\} u_y(x, \hat{\theta} - \phi(x, \hat{\theta})) - (\text{Term 3})}{[u_y(x, \hat{\theta} - \phi(x, \hat{\theta}))]^2},$$

where

$$\text{Term 1} = u_{yy}(x, \hat{\theta} - \phi(x, \hat{\theta})) \times [1 - \phi_{\hat{\theta}}(x, \hat{\theta})],$$

$$\text{Term 2} = u_{xy}(x(\hat{\theta}), \hat{\theta} - \phi(x(\hat{\theta}), \hat{\theta}))\dot{x}(\hat{\theta}) + u_{yy}(x(\hat{\theta}), \hat{\theta} - \phi(x(\hat{\theta}), \hat{\theta})) \times [1 - \phi_x(x(\hat{\theta}), \hat{\theta})\dot{x}(\hat{\theta}) - \phi_{\hat{\theta}}(x(\hat{\theta}), \hat{\theta})],$$

$$\text{Term 3} = [u_y(x, \hat{\theta} - \phi(x, \hat{\theta})) - u_y(x(\hat{\theta}), \hat{\theta} - \phi(x(\hat{\theta}), \hat{\theta}))] u_{yy}(x, \hat{\theta} - \phi(x, \hat{\theta})) \times [1 - \phi_{\hat{\theta}}(x, \hat{\theta})].$$

Notice that Term 3 vanishes at $x = x(\hat{\theta})$. Evaluating the second-order condition at $x = x(\hat{\theta})$ and $y(\theta) = \theta - \phi(x(\theta), \theta)$, together with the first-order condition, $\phi_{\hat{\theta}}(x(\theta), \theta) = 0$, to obtain

$$\phi_{\hat{\theta}\hat{\theta}}(x(\theta), \theta) = -\frac{N(x(\theta), y(\theta))\dot{x}(\theta)}{u_y(x, \hat{\theta} - \phi(x, \hat{\theta}))}$$

because the expression for Term 1 less Term 2 is given by

$$\begin{aligned} & u_{yy}(x(\theta), y(\theta)) - u_{xy}(x(\theta), y(\theta))\dot{x}(\theta) - u_{yy}(x(\theta), y(\theta)) \times [1 - \phi_x(x(\theta), \theta)\dot{x}(\theta)] \\ &= -\{u_{xy}(x(\theta), y(\theta)) - u_{yy}(x(\theta), y(\theta)) \times MRS_{xy}(x(\theta), y(\theta))\}\dot{x}(\theta) \\ &= -N(x(\theta), y(\theta))\dot{x}(\theta) \leqslant 0. \end{aligned}$$

Therefore, I conclude that $\phi_{\hat{\theta}\hat{\theta}}(x(\theta), \theta) \leqslant 0$. This establishes the last assertion. □

Now, the individual rationality under the price schedule $t(\cdot)$ is guaranteed.

Proposition 1. *If $\langle x(\cdot), p(\cdot)\rangle$ is incentive compatible and individually rational then*

$$u(x(\theta), \theta - t(x(\theta))) \geqslant \bar{\pi}(\theta)$$

for every $\theta \in \Theta$.

Proof. Since $\theta \in \operatorname{argmax}[\phi(x(\theta), \hat{\theta}) \mid \hat{\theta} \in \Theta]$, it follows that

$$t(x(\theta)) = \max\left[\phi(x(\theta), \hat{\theta}) \mid \hat{\theta} \in \Theta\right] = \phi(x(\theta), \theta) = p(\theta).$$

Therefore,

$$u(x(\theta), \theta - t(x(\theta))) = u(x(\theta), \theta - p(\theta)) \geqslant \bar{\pi}(\theta).$$

This establishes the proposition. □

Now, I am ready to verify the consumer's utility maximization problem under the price schedule $t(\cdot)$.

Proposition 2. *If $\langle x(\cdot), p(\cdot)\rangle$ is incentive compatible and individually rational then*

$$\max[u(x, \theta - t(x)) \mid x \in X] = u(x(\theta), \theta - t(x(\theta)))$$

for every $\theta \in \Theta$.

Proof. We have observed that $r(\theta) = u(x(\theta), \theta - t(x(\theta))) - \bar{\pi}(\theta)$. It suffices to show that $r(\theta) = \max[u(x, \theta - t(x)) \mid x \in X] - \bar{\pi}(\theta)$. Firstly, since

$$\max[u(x, \theta - t(x)) \mid x \in X] - \bar{\pi}(\theta) - r(\theta) \geqslant u(x(\theta), \theta - t(x(\theta))) - \bar{\pi}(\theta) - r(\theta) = 0,$$

it follows that $\max[u(x, \theta - t(x)) \mid x \in X] - \bar{\pi}(\theta) \geqslant r(\theta)$. It remains to show the converse inequality. Consider any $x \in X$. By definition of $t(x)$, I see that $t(x) \geqslant \phi(x, \theta)$. Since $u_y(x, y) > 0$, it follows that $r(\theta) = u(x, \theta - \phi(x, \theta)) - \bar{\pi}(\theta) \geqslant u(x, \theta - t(x)) - \bar{\pi}(\theta)$. Since x was arbitrary, it follows that $r(\theta) \geqslant \max[u(x, \theta - t(x)) \mid x \in X] - \bar{\pi}(\theta)$. Combining two inequalities to obtain $\max[u(x, \theta - t(x)) \mid x \in X] - \bar{\pi}(\theta) = r(\theta)$.

Therefore, I obtain

$$\max[u(x, \theta - t(x)) \mid x \in X] = u(x(\theta), \theta - t(x(\theta))).$$

This establishes the proposition. □

The above proposition establishes the implementability of the decision rule via price function $t(\cdot)$, that is,

$$x(\theta) \in \operatorname{argmax}[u(x, \theta - t(x)) \mid x \in X].$$

Theorem 2 (Taxation Principle)**.** *If a direct revelation mechanism $\langle x(\cdot), p(\cdot)\rangle$ is incentive compatible and individually rational, then it is voluntarily implementable.*

Proof. Immediate from Propositions 1 to 2. □

Now, I have obtained the characterization result for the agent whose income is private information considerd in Roberts (1979).

Theorem 3. *Under Assumtion 1 (Normality), a direct revelation mechanism $\langle x(\cdot), p(\cdot)\rangle$ is incentive compatible and individually rational if and only if it is voluntarily implementable.*

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