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Abstract

We develop normative investigation of sharing international rivers. First, we propose the model of water problems in the situation where a river flows through several states with the possibility of water shortage. We derive claims problems from the water problems. We axiomatize the family of convex combinations of the proportional and the equal awards rules for water claims problems. Using a unique claim vector constrained by geographic factors of a watercourse and the majority voting rule, we demonstrate how to determine the legal and political agreement of water problems.

Keywords: international river; claims problems; axiomatization; proportional rules; equal awards rules; median voter theorem

JEL classification: D63; K32

1 Introduction

An international river is a transboundary watercourse through more than two states. The international rivers are managed by the following environmental law: the Helsinki Rules on the uses of the waters of international rivers (for short, the Helsinki Rules), and the United Nations Convention on the law of the non-navigational uses of international watercourses (for short, the United

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Nations Convention).[1] Furthermore, under international environmental law, international river management is done by international commissions whose members are the watercourse states involved.

We investigate legal and political agreements for sharing an international river among watercourse states. In particular, we are interested in how the benefits of the usage of an international river should be divided among the watercourse states that may suffer from water shortage.

The Helsinki Rules and the United Nations Convention play a very significant role in management of international rivers.[2] As stated in LeMarquand (1977), an international river is a common property resource shared among the basin states, but the property rights over the waters through each basin state are not well defined. This implies that the Coase theorem (Coase 1960) cannot be applied. There have been ongoing conflicts over transboundary waters, e.g., the Jordan River (Israel vs. Lebanon), the Euphrates River (Turkey vs. Syria), and the Indus River (India vs. Pakistan). However, international tensions are currently decreasing, through the international environmental law mentioned above.[3]

The motivation for this study steams from the fact that each watercourse state is entitled, within its territory, to a reasonable and equitable share in the beneficial uses of the waters of international rivers, e.g., the Helsinki Rules, Article IV and the United Nations Convention, Article 5. A "reasonable" principle is the principle of acceptable and appropriate uses of the entire river among the watercourse states. On the other hand, an "equitable" principle is based on egalitarianism of the exercise of rights over a watercourse by each watercourse state. The literature on international environmental law indicates that it is difficult to answer the following practical question: What kind of reasonable and equitable sharing scheme is useful for international river management? How can international commissions compromise among conflicts of claims to the commission members' benefits of the usage of waters? The present study sheds a light on these questions.

We develop normative investigation of sharing international rivers. In order to achieve our goal, we develop the model proposed by Ambec and Sprumont (2002). Although Ambec and Sprumont (2002) is the seminal work of economic analysis of water problems, this work may fail to capture two significant aspects in practice. First, their model describes no possibility of water shortage. Water shortage in downstream states is a major reason for international

[1] The Helsinki Rules are adopted by the International Law Association in 1966, and the United Nations Convention is formulated by the International Law Commission in 1997.

[2] More than 260 river basins are international river basins.

[3] For instance, competition for the waters of the Nile River between Egypt, Sudan, Ethiopia, and the Lake Victoria basin states has been replaced by cooperation through the United Nations Convention.

conflicts over transboundary waters.[4] Second, a unique outcome of the river problems described in their paper does not allow for any consideration of reasonable and equitable sharing scheme. This is because the unique outcome is associated with potential utilization of waters among the watercourse states. However, Article V of the Helsinki Rules and Article 6 of the United Nations Convention state that many factors other than potential utilization of waters are to be considered in the reasonable and equitable use of waters. From the two aspects, we describe the model of water problems in the situation where a watercourse flows through several states with the possibility of water shortage, and analyze "claims problems" derived from the water problems.[5] The water claims problem is to determine how the watercourse states should share the welfare among themselves on the basis of their claims. In the water claims problems, we investigate an axiomatic analysis using reasonable and equitable sharing scheme.

We propose various properties of reasonable and equitable sharing schemes for water claims problems. For the reasonable sharing scheme, we propose "efficiency", "continuity", and "reallocation-proofness". *Efficiency* requires that the value of the maximal welfare should be distributed among the watercourse states. *Continuity* requires that a small change in the claims should lead to a small change in the outcome chosen by a rule. *Reallocation-proofness* requires that watercourse states should have no incentive to transfer their claims among themselves. For the equitable sharing scheme, on the other hand, we propose "anonymity". *Anonymity* requires that the outcome chosen by a rule should depend only on the list of claims.

Using all the properties mentioned above, we axiomatize the family of convex combinations of the *proportional* and the *equal awards rules* for water claims problems. In the present study, this family is referred to as the "α-egalitarian proportional rule." Here, the share ratio $\alpha \in [0, 1]$ is the weight on the proportional rule, and the share ratio $1-\alpha$ is the weight on the equal awards rule. The proportional and equal awards rules are the most popular rules for claims problems in practice. As stated in Moulin (1987), the equal awards rule is the most egalitarian sharing method, and the proportional rule is the least egalitarian sharing method. The α-egalitarian proportional rule is the family of rules that compromise between the two focal rules. The α-egalitarian proportional rule is the only family of rules that satisfy the reasonable and equitable sharing schemes mentioned above.[6]

[4] The 21st century is said to be "the age of water war". For the detail, see Postel (2006).

[5] For the literature of the general class of claims problems, for instance, see O'Neill (1982), Aumann and Maschler (1985), Chun (1988), Thomson (2003), Moreno-Ternero (2006), and Ju, Miyagawa, and Sakai (2007). For the literature of the subclass of claims problems applied to water problems, for instance, see Ansink and Weikard (2012). For our comments on Ansink and Weikard (2012), see Section 5.

[6] For other characterizations of the family of convex combinations of the proportional

Next, we specify a claim of the watercourse states by considering lower bounds and upper bounds of waters, which are potentially utilized by each state. We call it the "constrained claim vector" since utilization of waters is constrained by geographic factors of an international river. A unique constrained claim vector is shown. Under the potential utilization of waters among the states, the outcome chosen by the α-egalitarian proportional rule is the convex combination of the constrained claim vector and the equal division of the whole constrained claims.

Using the constrained claim vector and the majority voting rule, we consider how to determine the share ratio α. In practice, international river management by an international commission, which consists of the watercourse states involved, is recommended by international environmental law, e.g., the Danube Commission. We consider the situation where in the commission each watercourse state votes on a share ratio $\alpha \in [0, 1]$. Each state's preference over the interval of a share ratio is *single-peaked*. By the Median Voter Theorem, we can determine the share ratio $\alpha = 1$ or $\alpha = 0$. Therefore, the constrained claim vector is chosen as the legal and political agreement of water problems if the states who prefer $\alpha = 1$ to $\alpha = 0$ consist of a majority. Otherwise, the equal award division is chosen as the legal and political agreement of water problems.

The rest of this paper is organized as follows. In Section 2, we introduce a model of water problems. In Section 3, we introduce water claims problems derived from the water problems, and axiomatize the family of the convex combinations of the proportional and the equal awards rules. In Section 4, we show a unique constrained claim vector, and demonstrate how to determine the share ratio under the potential utilization of waters among the watercourse states. In Section 5, we discuss several related papers to our study except for Ambec and Sprumont (2002), and an open question. In the Appendix, we show the logical independence of the axioms proposed, and the unique existence of a constrained claim vector.

2 A model of water problems

We develop the model proposed by Ambec and Sprumont (2002) by considering water shortage structure. The difference between their model and our model is discussed in the last paragraph of this section.

Let $\mathcal{U} \subseteq \mathbb{N}$ be a universe of agents with at least two agents.[7] We denote by $N \subseteq \mathcal{U}$ a finite non-empty subset of $\mathcal{U}$, and $n \equiv |N|$.

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and the equal awards rules, for instance, see Moulin (1987) and Giménez-Gómez and Peris (2014). Note that these papers are not related to water problems.

[7] We use $\subseteq$ for weak set inclusion, and $\subset$ for strict set inclusion.

Imagine a line divided into n segments indexed by $i = 1, 2, \cdots, n$ with $n \geq 2$. Each segment i corresponds to state i. A watercourse flows from state 1 (i.e. the most upstream state) to state n (i.e. the most downstream state). We say that j is **downstream** of state i if $j > i$. On the other hand, we say that state j is **upstream** of state i if $j < i$. The set of states is denoted by N.

Each state $i \in N$ has a source of water as the **endowment**. We denote by e_i the quantity of water at state i's *endowment*. For each $i \in N$, let $e_i > 0$. The river picks up quantity of water along its course: The quantity of water is increased by e_i when the river flows through state i. Water is a private good. Each state $i \in N$ consumes x_i units of water. Each state i needs at minimal amounts $\bar{x}_i$ units of water to save people. The amount $\bar{x}_i$ is referred to as the **essential water consumption** of i. The *essential water consumption* of each state $i \in N$ is *feasible* if for each $i \in N$,

$$\sum_{k=1}^{i} \bar{x}_k \leq \sum_{k=1}^{i} e_k.$$

We assume that the states that are downstream of state 1 suffer from water shortage if they cannot utilize waters of e_1: For each $i, j \in N \backslash \{1\}$ with $i \leq j$

$$\sum_{k=i}^{j} e_k < \sum_{k=i}^{j} \bar{x}_k.$$

This assumption together with the feasibility condition mentioned above mean that the source of water at state 1 is crucial for the states that are downstream of state 1.

State i's benefit is derived from its water consumption. Let state i's **benefit function** be given by $\pi_i : \mathbb{R}_+ \rightarrow \mathbb{R}$. The benefit function is *strictly increasing, strictly concave,* and *differentiable at each* $x_i > 0$. Assume that its derivative $\pi_i'(x_i)$ goes to infinity as x_i tends to zero. **Extraction cost of water per unit**, denoted $\bar{\mathbf{c}}$, is *constant.*

For each state $i \in N$, the marginal benefit with respect to the *essential water consumption* is larger than the *marginal cost*: $\pi_i'(\bar{x}_i) > \bar{\mathbf{c}}$. Furthermore, for each pair $\{i, j\}$ such that $i, j \in N$ and $i > j$, and for each pair $\{x_j, \bar{x}_j\}$ such that $x_j > \bar{x}_j$, there is a positive ϵ such that $\pi_i'(\bar{x}_i + \epsilon) > \pi_j'(x_j)$. This assumption may be interpreted as follows: Each state suffering from a water shortage wants more water than its upstream states that do not suffer from a water shortage.

Money is available in unbounded quantity to perform side-payments. States value money and water. State i's **utility**, from consuming x_i units of water and receiving a net money transfer t_i, is given by $u_i : \mathbb{R}^2 \rightarrow \mathbb{R}$ such that $u_i(x_i, t_i) = \pi_i(x_i) - \bar{\mathbf{c}} \cdot x_i + t_i$.

We refer to $w \equiv (N, e, \bar{x}, \pi, \bar{\mathbf{c}})$, where $e = (e_1, e_{2,} \cdots, e_n)$, $\bar{x} = (\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_n)$ and $\pi = (\pi_1, \pi_2, \cdots, \pi_n)$, as a **water problem** on $\mathcal{U}$. Let $\mathcal{W}$ be the set of all the water problems on $\mathcal{U}$.

An **allocation** is a vector $(x, t) = (x_1, \cdots, x_n, t_1, \cdots, t_n) \in \mathbb{R}^n_+ \times \mathbb{R}^n$ satisfying the *feasibility* constraints:

$$\sum_{i \in N} t_i \leq 0, \quad x_j \geq \bar{x}_j \text{ for each } j \in N,$$
$$\sum_{i=1}^{j} x_i \leq \sum_{i=1}^{j} e_i \quad \text{for } j = 1, \cdots, n.$$

An allocation (x^*, t^*) is **efficient** if and only if it maximizes the sum of all states' benefits and wastes no money.

We assume that the *efficient* amount of water consumption is greater than *essential* amount of water consumption: For each $i \in N$,

$$x_i^* > \bar{x}_i.$$

If this assumption does not hold, then an efficient allocation makes no sense in practice.

Proposition 1 *For each water problem $w \in \mathcal{W}$, there is a unique efficient water consumption.*

Proof. Consider the following problem (P):

$$(P) \quad : \quad \max_{x,t} \left(\sum_{i \in N} (\pi_i(x_i) - \bar{\mathbf{c}} \cdot x_i) + \sum_{i \in N} t_i \right)$$
$$s.t. \sum_{i \in N} t_i \leq 0,\ x_j \geq \bar{x}_j,\ \sum_{i=1}^{j} x_i \leq \sum_{i=1}^{j} e_i \quad \text{for } j = 1, \cdots, n.$$

Let $\mathcal{L}$ be the Lagrangian derived from Problem (P), namely

$$\mathcal{L} \equiv -\sum_{j \in N} \pi_j(x_j) + \bar{\mathbf{c}} \sum_{j \in N} x_j + \sum_{j \in N} \lambda_j (-x_j + \bar{x}_j) + \sum_{j \in N} \gamma_j \sum_{k=1}^{j} (x_k - e_k).$$

By the Kuhn-Tucker condition, a pair (x^*, t^*) is an optimal solution for problem

(P) if and only if for each $j \in N$,

$$\lambda_j \geq 0, \gamma_j \geq 0, \; -x_j^* + \bar{x}_j \leq 0, \; \sum_{k=1}^{j} (x_k^* - e_k) \leq 0,$$

$$\lambda_j \left(-x_j^* + \bar{x}_j\right) = 0, \gamma_j \sum_{k=1}^{j} (x_k^* - e_k) = 0, \text{ and } \; -\pi_j'(x_j^*) + \bar{\mathbf{c}} - \lambda_j + \sum_{k=j}^{n} \gamma_k = 0.$$

Since $\pi_j'(0) \to \infty$, for each $j \in N$ $x_j^* > 0$. For each $j \in N$, since $x_j^* > \bar{x}_j$, $\lambda_j = 0$ and $\pi_j'(x_j^*) = \bar{\mathbf{c}} + \sum_{k=j}^{n} \gamma_k$.

For each $j \in N$, let $\alpha_j \equiv \pi_j'(x_j^*)$. Since for each $j \in N$ $\gamma_j \geq 0$, for each pair $\{i, i'\}$ such that $i, i' \in N$ and $i < i'$ $\alpha_i \geq \alpha_{i'}$. Let $i_1 \equiv \min\{i \in N : \gamma_i > 0\}$, $i_2 \equiv \min\{i \in N : i > i_1, \gamma_i > 0\}$, $\cdots$, $i_K \equiv \min\{i \in N : i > i_{K-1}, \gamma_i > 0\}$, where $i_K = n$. We have the partition of N given by $N_1 \equiv \{1, \cdots, i_1\}$, $N_2 \equiv \{i_1 + 1, \cdots, i_2\}$, $\cdots N_K \equiv \{i_{K-1} + 1, \cdots, i_K\}$.

For each $i \in N_k$ $(k = 1, \cdots, K)$, let $\gamma_i \equiv \gamma_{i_k} > 0$. Since for each i, j such that $i, j \in N \backslash \{1\}$ and $i \leq j$ $\sum_{k=i}^{j} e_k < \sum_{k=i}^{j} x_k^*$, we have that $N_1 = N$, $\sum_{i \in N} (x_i^* - e_i) = 0$, $\gamma_n > 0$, and for each $i \neq n$ $\gamma_i = 0$. Since for each $i \in N$ $\pi_i'(x_i^*) = \bar{\mathbf{c}} + \gamma_n < \pi_i'(\bar{x}_i)$ and $\pi_i'(\bar{x}_i) > \bar{\mathbf{c}}$ by the assumption, there is a positive number γ_n such that $\gamma_n < \min_{i \in N} \pi_i'(\bar{x}_i) - \bar{\mathbf{c}}$. Therefore, there is a unique solution for problem (P). ■

We point out the difference between the Ambec and Sprumont's model and our model as follows: We deal with a situation where downstream states of state 1 may suffer from water shortage. This is because water sources of the states that are downstream of state 1 have insufficient quantity of waters. Although Ambec and Sprumont account contributions by states in their model, their model describes no possibility of water shortage. In the real world, however, conflicts over transboundary waters among watercourse states often arise from water shortage in downstream states. Our model allows for a simple consideration of this kind of water shortage problems.

3 Claims problems among watercourse states

Next, we analyze how to split the welfare among the watercourse states by considering reasonable and equitable use of waters. For this purpose, we introduce **water claims problems**. A water claims problem is a *claims problem* (O'Neill 1982; Aumann and Maschler 1985)[8] derived from a water problem.

[8] Claims problems deal with the situation where the liquidation value of a bankrupt firm has to be allocated among its creditors, but there is not enough to honor the claims of all creditors. The problem is to determine how the creditors should share the liquidation value.

Let E be the sum of benefits of all the states in an efficient allocation (x^*, t^*), that is,

$$E \equiv \sum_{i \in N} (\pi_i(x_i^*) - \bar{\mathbf{c}} \cdot x_i^*) .$$

Let $b \in \mathbb{R}^N_{++}$ be the corresponding benefit profile on an efficient allocation (x^*, t^*), that is, for each $i \in N$ $b_i = \pi_i(x_i^*) - \bar{\mathbf{c}} \cdot x_i^*$. Note that for each $i \in N$ $b_i > 0$ since $\pi_i'(x_i^*) > \bar{\mathbf{c}}$ (see the proof of Proposition 1). We call b an **efficient benefit**.

Fix an arbitrary water problem $w \in \mathcal{W}$. Let E be the **estate** derived from the water problem w, which is the welfare to be distributed among the states (or claimants): $E = b_1 + b_2 + \cdots + b_n$. Let c_i be state i's **claim** (or right) against the estate E, that is, each state $i \in N$ claims the amount c_i. For $S \subseteq N$, let $c_S \equiv \sum_{i \in S} c_i$. We do not impose the condition $E \leq c_N$.

We assume that for each $i \in N$ $c_i \geq \min_{j \in N} b_j$. This assumption says that each state can claim at least the smallest *efficient benefit* among all the states. Note that it does not require that each state claims at least its own *efficient benefit*, but the smallest *efficient benefit* among all the states.

For each $w \in \mathcal{W}$, a **water claims problem** is a pair $(c, E) \in \mathbb{R}^{n+1}_{++}$. Let $\mathcal{P}$ be the set of water claims problems on $\mathcal{W}$. For each water problem $w \in \mathcal{W}$, let $X(w)$ be the set of allocations: $X(w) \equiv \{x \in \mathbb{R}^N_+ : \sum_{i \in N} x_i \leq E\}$. For each water problem $w \in \mathcal{W}$, an **allocation rule** (simply, a **rule**) is a mapping, denoted φ, that associates with each water claims problem $(c, E) \in \mathcal{P}$ an allocation $x \in X(w)$.

We are interested in rules based on the *reasonable* and *equitable* sharing principles stated in the international rules for transboundary watercourses. For instance, Article V of the Helsinki Rules and Article 6 of the United Nations Convention state that a *reasonable* and *equitable* share is to be determined in the light of all relevant factors in each particular case. These international rules state that relevant factors that are to be considered include, but are not limited to,

Relevant factors stated in Article V of the Helsinki Rules and Article 6 of the United Nations Convention

(i): *Geographic factors of the watercourse, including, in particular, the contribution of water by each watercourse state;*
(ii): *The practicability of compensation to one or more of the co-watercourse states as a means of adjusting conflicts among users;*
(iii): *The economic and social needs of each watercourse state.*

The factor (i) is included in formalization of water problems that are mentioned in Section 2. The factor (ii) is considered in *rules* since they are monetary compensation. In order to catch a light on the factor (iii), we consider what is a *reasonable* and *equitable* use of waters. Unfortunately, in the Helsinki Rules and the United Nations Convention, what is a reasonable and equitable use is not defined explicitly. In order to discuss *reasonable* and *equitable* principles, we borrow from the literature of international environmental law, e.g., Birnie, Boyle, and Redgwell (2009).

A *reasonable* principle is a principle of acceptable and appropriate uses of the entire river among the watercourse states. In Birnie, Boyle, and Redgwell (2009), this principle is the basis of objective rules for management of the entire river when the uses of the waters of each state do not disturb other states' rights to the waters.

Efficiency requires that for each water claims problem the whole value of the *estate* should be distributed among the states. **Continuity** requires that a small change in the claims of each water claims problem should not lead to a large change in the outcome chosen by a rule.

Efficiency (Eff): For each $w \in \mathcal{W}$, and each $(c, E) \in \mathcal{P}$, $\sum_{i \in N} \varphi_i(c, E) = E$.

Continuity (Cont): For each $w \in \mathcal{W}$, and each sequence $\{(c^k, E)\}$ of elements of $\mathcal{P}$, if $c^k \to c^*$, then $\varphi(c^k, E) \to \varphi(c^*, E)$.

The following property says that the states never benefit from transferring their claims among themselves. In this sense, this property is reasonable.

Reallocation-proofness (RAP): For each $w \in \mathcal{W}$, each $(c, E) \in \mathcal{P}$, and $T \subset N$ with $T \neq \emptyset$,

$$\sum_{i \in T} \varphi_i(c, E) = \sum_{i \in T} \varphi_i((c'_i)_{i \in T}, (c_i)_{i \in N \setminus T}, E),$$

where $((c'_i)_{i \in T}, (c_i)_{i \in N \setminus T}, E) \in \mathcal{P}$ such that $c_T = c'_T$.

Reallocation-proofness is a standard property in the literature on claims problems. For instance, see Thomson (2003) and Ju, Miyagawa, and Sakai (2007).

An *equitable* principle is a principle of acceptable and appropriate uses of the waters in each state. In Birnie, Boyle, and Redgwell (2009), this principle is considered to be the basis for common law governing assignment of rights over international rivers among states. In particular, the *equitable* principle is based on providing equal opportunity of access to a river by each state.

The following property requires that the outcome chosen by a rule should depend only on the list of claims, not on who holds them. It is an elementary principle of egalitarianism.

Anonymity (AN): For each $w \in \mathcal{W}$, each $(c, E) \in \mathcal{P}$, each permutation $\sigma : N \to N$ and each $i \in N$, $\varphi_i(c, E) = \varphi_{\sigma(i)}(c_\sigma, E)$, where $c_\sigma \equiv (c_{\sigma(i)})_{i \in N}$.

The **proportional rule** is the commonly used rule for claims problems in practice. For each $w \in \mathcal{W}$, each $(c, E) \in \mathcal{P}$, and each $i \in N$, it is defined by

$$PR_i(c, E) \equiv \frac{c_i}{c_N} E.$$

The **equal awards rule** is one of the most important rules for claims problems in the literature. For each $w \in \mathcal{W}$, each $(c, E) \in \mathcal{P}$, and each $i \in N$, it is defined by

$$EA_i(c, E) \equiv \frac{E}{n}.$$

We consider the family of convex combinations of the *proportional* and the *equal awards rules.*

Let $\alpha \in [0, 1]$. For each $w \in \mathcal{W}$, and each $(c, E) \in \mathcal{P}$, the **α-egalitarian proportional rule**, denoted φ^α, is defined by

$$\varphi^\alpha(c, E) \equiv \alpha PR(c, E) + (1 - \alpha) EA(c, E).$$

We characterize the α-egalitarian proportional rule for water claims problems as follows:

Theorem 1 *For each $w \in \mathcal{W}$ such that $n \geq 3$, and each $(c, E) \in \mathcal{P}$, a rule satisfies efficiency, anonymity, continuity, and reallocation-proofness if and only if there is $\alpha \in [0, 1]$ such that the rule is the α-egalitarian proportional rule.*

Proof. If there is $\alpha \in [0, 1]$ such that a rule is the α-egalitarian proportional rule φ^α, then it is clear that φ^α satisfies the four properties. We show that if a rule satisfies efficiency, anonymity, continuity, and reallocation-proofness then there is $\alpha \in [0, 1]$ such that the rule is the α-egalitarian proportional rule. Let $N = \{1, 2, \cdots, n\}$ with $n \geq 3$ and $c \equiv (c_1, c_2, \cdots, c_n)$ be given. Let $m \equiv \min_{j \in N} b_j$.

Claim 1 For each $i \in N$, $\varphi_i(c, E) = \frac{c_i}{c_N} E - \frac{1}{c_N} (nc_i - c_N)\, g(c_N, E)$, where $g(c_N, E) \equiv \varphi_1(m, c_N - (n-1)m, m, \cdots, m, E)$.

Let φ be a rule satisfying the five axioms. Now let $c' \equiv (c_1 + c_2 - m, m, c_3, c_4, \cdots, c_n)$. Note that $c_1 + c_2 - m \geq m$. We have

$$\varphi_1(c, E) + \varphi_2(c, E) \overset{\textbf{RAP}}{=} \varphi_1(c', E) + \varphi_2(c', E). \tag{1}$$

Let $c'' \equiv (c_1, c_{N\backslash\{1\}} - (n-2)m, m, \cdots, m)$, where for each $N' \subseteq N$ $c_{N\backslash N'} \equiv \sum_{j\in N\backslash N'} c_j$, and $c_N \equiv \sum_{j\in N} c_j$. Note that $c_{N\backslash\{1\}} - (n-2)m \geq (n-1)m - (n-2)m = m$. Let $N' \equiv N\backslash\{1\}$. We have

$$\sum_{i\in N'} \varphi_i(c, E) \overset{\textbf{RAP}}{=} \sum_{i\in N'} \varphi_i(c'', E) \tag{2}$$

By this observation,

$$\varphi_1(c, E) \overset{\textbf{Eff}}{=} \varphi_1(c'', E). \tag{3}$$

Similarly, for each $i \in N$

$$\begin{aligned}
\varphi_i(c, E) &\overset{(3),\textbf{AN}}{=} \varphi_1(c_i, c_{N\backslash\{i\}} - (n-2)m, m, \cdots, m, E), \qquad (4)\\
\varphi_1(c', E) &\overset{(3)}{=} \varphi_1(c_1 + c_2 - m, c_{N\backslash\{1,2\}} - (n-3)m, m, \cdots, m, E), \text{ and}\\
\varphi_2(c', E) &\overset{(3),\textbf{AN}}{=} \varphi_1(m, c_N - (n-1)m, m, \cdots, m, E).
\end{aligned}$$

Note that $c_{N\backslash\{1,2\}} - (n-3)m \geq m$ and $c_N - (n-1)m \geq m$.

We have that

$$\begin{aligned}
&\varphi_1(c_1, c_{N\backslash\{1\}} - (n-2)m, m, \cdots, m, E)\\
&+\varphi_1(c_2, c_{N\backslash\{2\}} - (n-2)m, m, \cdots, m, E) \qquad (5)\\
&\overset{(1),(4)}{=} \varphi_1(c_1 + c_2 - m, c_{N\backslash\{1,2\}} - (n-3)m, m, \cdots, m, E)\\
&+\varphi_1(m, c_N - (n-1)m, m, \cdots, m, E).
\end{aligned}$$

Let $N' \subset N$, and $f : \mathbb{R}^3 \to \mathbb{R}$ and $g : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$\begin{aligned}
f(c_{N'}, c_N, E) \equiv\ & \varphi_1(c_{N'} - (|N'| - 1)m, c_{N\backslash N'} - (|N\backslash N'| - 1)m, m, \cdots, m, E)\\
& -\varphi_1(m, c_N - (n-1)m, m, \cdots, m, E) \qquad (6)
\end{aligned}$$

and

$$g(c_N, E) \equiv \varphi_1(m, c_N - (n-1)m, m, \cdots, m, E). \tag{7}$$

We have that for all c_1, c_2, and E,

$$f(c_1, c_N, E) + f(c_2, c_N, E) \overset{(5),(6)}{=} f(c_1 + c_2, c_N, E).$$

Since $n \geq 3$, f is additive with respect to its first argument for each c_N and E. By **Cont**, f is continuous. Applying a theorem on Cauchy's equation (Aczél 1966) to f, there exists a continuous function $h : \mathbb{R}^2 \to \mathbb{R}$ such that

$$f(c_i, c_N, E) = c_i h(c_N, E). \tag{8}$$

By substituting (8) to (6),

$$\begin{aligned} c_i h(c_N, E) &= \varphi_1(c_i, c_{N\setminus\{i\}} - (n-2)m, m, \cdots, m, E) - g(c_N, E) \\ &\overset{(4)}{=} \varphi_i(c, E) - g(c_N, E), \end{aligned}$$

which implies

$$\varphi_i(c, E) = c_i h(c_N, E) + g(c_N, E). \tag{9}$$

Since $c_N h(c_N, E) + n g(c_N, E) \overset{\mathbf{Eff}}{=} E$,

$$h(c_N, E) = \frac{E - n g(c_N, E)}{c_N}. \tag{10}$$

For each $i \in N$,

$$\varphi_i(c, E) \overset{(9),(10)}{=} \frac{c_i}{c_N} E - \frac{1}{c_N} (n c_i - c_N) g(c_N, E). \tag{11}$$

Claim 2 There is $\alpha \in [0, 1]$ such that $g(c_N, E) = (1-\alpha)\frac{E}{n}$.

First, we claim that for each $i \neq 2$,

$$\varphi_2(m, c_N - (n-1)m, m, \cdots, m, E) \geq \varphi_i(m, c_N - (n-1)m, m, \cdots, m, E). \tag{12}$$

Note that $c_N - (n-1)m \geq nm - (n-1)m = m$.

Suppose not. By this supposition together with **AN**, for each $i \neq 2$,

$$\varphi_2(m, c_N - (n-1)m, m, \cdots, m, E) < \varphi_i(m, c_N - (n-1)m, m, \cdots, m, E). \tag{13}$$

By **AN**, for each pair $\{i', j'\}$ such that $i', j' \in N$ with $i', j' \neq 2$,

$$\begin{aligned} \varphi_{i'}(m, c_N - (n-1)m, m, \cdots, m, E) &= \varphi_{j'}(m, c_N - (n-1)m, m, \cdots, m, E) \\ &\equiv r. \end{aligned} \tag{14}$$

By **Eff** together with (13) and (14),

$$E - (n-1)r < r,$$

or equivalently, $E < nr$. Since $\varphi(\cdot) \geq 0$, $r \in (\frac{E}{n}, \frac{E}{n-1}]$, which implies that there is $t \in [0, 1)$ such that

$$\varphi_1(m, c_N - (n-1)m, m, \cdots, m, E) = t\frac{E}{n} + (1-t)\frac{E}{n-1} \overset{(7)}{=} g(c_N, E).$$

By Claims 1 and 2, for each $i \in N$, there is $t \in [0, 1)$ such that

$$\begin{aligned}\varphi_i(c, E) &= \frac{c_i}{c_N}E - \frac{1}{c_N}(nc_i - c_N)\left(t\frac{E}{n} + (1-t)\frac{E}{n-1}\right)\\ &= \frac{c_i}{c_N}E - \left(\frac{t}{n} + \frac{1-t}{n-1}\right)\frac{nc_i - c_N}{c_N}E.\end{aligned}$$

Since $PR(c, E)$ satisfies the four axioms, there is $t \in [0, 1)$ such that for each $(c, E) \in \mathcal{P}$ $\varphi(c, E) = PR(c, E)$. However, the equation $\frac{t}{n} + \frac{1-t}{n-1} = 0$ implies that $t = n$, which is impossible.

Next, we claim that there is $\alpha \in [0, 1]$ such that $g(c_N, E) = (1-\alpha)\frac{E}{n}$. By **Eff** together with (12) and (14),

$$E - (n-1)r \geq r,$$

or equivalently, $E \geq nr$. Since $\varphi(\cdot) \geq 0$, $r \in [0, \frac{E}{n}]$, which implies that there is $\alpha \in [0, 1]$ such that

$$\varphi_1(m, c_N - (n-1)m, m, \cdots, m, E) = (1-\alpha)\frac{E}{n} \overset{(7)}{=} g(c_N, E).$$

Claim 3 There is $\alpha \in [0, 1]$ such that $\varphi(c, E) = \alpha PR(c, E) + (1-\alpha)EA(c, E)$.

By Claims 1 and 2, for each $i \in N$, there is $\alpha \in [0, 1]$ such that

$$\begin{aligned}\varphi_i(c, E) &= \frac{c_i}{c_N}E - \frac{1}{c_N}(nc_i - c_N)(1-\alpha)\frac{E}{n}\\ &= \frac{c_i}{c_N}E - \frac{c_i}{c_N}(1-\alpha)E + (1-\alpha)\frac{E}{n}\\ &= \alpha\frac{c_i}{c_N}E + (1-\alpha)\frac{E}{n},\end{aligned}$$

which completes the proof. ■

For checking the logical independence of the four axioms, see Appendix A. Furthermore, we remark on the number of basin states, and the difference between Chun (1988) and the present study. First, in the real world, many international rivers flow through more than three states. For instance, Human Development Report (2006) by United Nations Development Programme states that 14 states share the Danube, 11 the Nile and the Niger, and 9 the Amazon. Therefore, the assumption that $n \geq 3$ appearing in Theorem 1 is justified. Next, Theorem 1 appearing in Chun (1988) shows Claim 1 that is mentioned above in the case of $m = 0$. In the present model, we deal with the situation where $m > 0$.

4 Single-peakedness of voting in the commission

Next, we specify each state's claim (or right) in the context of international law doctrines. *Article V of the Helsinki Rules and Article 6 of the United Nations Convention* state that other relevant factors that are to be considered include, but are not limited to,

(iv): *Existing and potential utilization of the watercourse.*

We focus on factor (iv). Based on Article 6 of the United Nations Convention, a **constrained claim** of state i is defined to be a benefit from its potential utilization of the international river. Let $b^* \equiv (b_1^*, b_2^*, \cdots, b_n^*)$ be a *constrained claim vector*, where b_i^* is state i's constrained claim. Ambec and Sprumont (2002) formalized a constrained claim vector by using the notions of the **core lower bounds** and the **aspiration upper bounds**.[9] The core lower bound is inspired from an international law doctrine called **absolute territorial sovereignty**. This lower bound property requires that no coalition should get less than the welfare attainable by the water the coalition controls. The *aspiration upper bound*, on the other hand, is inspired from another international law doctrine called **unlimited territorial integrity**. This upper bound property requires that no coalition should get a welfare higher than what it can achieve in the absence of the remaining states.

Let U_i be the set of upstream states of state i, namely $U_i \equiv \{j \in N : j < i\}$ with $U_1 = \emptyset$. Let $U_i^0 \equiv U_i \cup \{i\}$. A coalition $S \subseteq N$ is *consecutive* if $k \in S$ whenever $i, j \in S$ and $i < k < j$. Let $\mathcal{P}_S$ be the unique coarsest partition *of* S into consecutive components.

For each coalition $S \subseteq N$, let $z^*(S) \in \mathbb{R}_+^S$ be a consumption plan of waters under absolute territorial sovereignty that maximizes $\sum_{i \in S} (\pi_i(z_i) - \bar{\mathbf{c}} \cdot z_i)$ subject to the constraints: (a) for each $T \in \mathcal{P}_S$ and each $j \in T$, $\sum_{i \in U_j^0 \cap T}(z_i - e_i) \leq 0$; (b) for $T \in \mathcal{P}_S$ such that $1 \in T$ and for each $i \in T$, $z_i \geq \bar{x}_i$, and for $T' \in \mathcal{P}_S$ such that $1 \notin T'$ and for each $i \in T'$, $z_i \geq 0$. Condition (a) is the water consumption feasibility of coalition S under absolute territorial sovereignty. Condition (b) says that under absolute territorial sovereignty since the members of the consecutive coalition $T \in \mathcal{P}_S$ including state 1 enjoy the source of water at state 1, they consume at least the essential waters. This condition also says that water consumptions of the members of coalition S are non-negative. We assume that for each $i \in T$ such that $T \in \mathcal{P}_S$ and $1 \in T$, $z_i^*(S) > \bar{x}_i$. If this assumption does not hold, then the core lower bounds make no sense

[9] A constrained claim vector is referred to as the *downstream incremental distribution* in Ambec and Sprumont (2002). Note that Ambec and Sprumont do not deal with claims problems derived from the water problems under consideration.

in practice. We can verify easily that for each water problem $w \in \mathcal{W}$ and each $S \subseteq N$, there is a unique consumption plan under absolute territorial sovereignty $z^*(S) \in \mathbb{R}^S_+$.[10]

For each coalition $S \subseteq N$, let $z^{**}(S) \in \mathbb{R}^S_+$ be a consumption plan of waters under unlimited territorial integrity that maximizes $\sum_{i\in S}(\pi_i(z_i) - \bar{\mathbf{c}} \cdot z_i)$ subject to the constraints: for each $j \in S$, (c) $\sum_{i \in U_j^0 \cap S} z_i \leq \sum_{i\in U_j^0} e_i$, and (d) $z_j \geq \bar{x}_j$. Condition (c) is the water consumption feasibility of coalition S under unlimited territorial integrity. Condition (d) says that under unlimited territorial integrity since the members of coalition S always enjoy the source of water at state 1, they consume at least the essential waters. We assume that for each $i \in S$, $z_i^{**}(S) > \bar{x}_i$. If this assumption does not hold, then the aspiration upper bounds make no sense in practice. We can verify easily that for each water problem $w \in \mathcal{W}$ and each $S \subseteq N$, there is a unique consumption plan under unlimited territorial integrity $z^{**}(S) \in \mathbb{R}^S_+$.[11]

An n-dimensional vector $b = (b_1, b_2, \cdots, b_n)$ satisfies the *core lower bounds* if for each $S \subseteq N$ $\sum_{i\in S} b_i \geq \sum_{i\in S}(\pi_i(z_i^*(S)) - \bar{\mathbf{c}} \cdot z_i^*(S))$. On the other hand, an n-dimensional vector $b = (b_1, b_2, \cdots, b_n)$ satisfies the *aspiration upper bounds* if for each $S \subseteq N$ $\sum_{i\in S} b_i \leq \sum_{i\in S}(\pi_i(z_i^{**}(S)) - \bar{\mathbf{c}} \cdot z_i^{**}(S))$.

The definition of a constrained claim vector is due to Ambec and Sprumont (2002).

Definition 1 ***(Constrained claim vector)*** *For each water problem $w \in \mathcal{W}$, a constrained claim vector is an n-dimensional vector satisfying the core lower bounds and the aspiration upper bounds.*

The following theorem shows the unique existence of a *constrained claim vector.*

Theorem 2 *For each water problem $w \in \mathcal{W}$, there exists a unique constrained claim vector $b^* \in \mathbb{R}^n_{++}$: For each $w \in \mathcal{W}$ and each $i \in N$,*

$$b_i^* = \sum_{j\in U_i^0} \big(\pi_j(z_j^*(U_i^0)) - \bar{\mathbf{c}} \cdot z_j^*(U_i^0)\big) - \sum_{j\in U_i} \big(\pi_j(z_j^*(U_i)) - \bar{\mathbf{c}} \cdot z_j^*(U_i)\big) > 0$$

or, equivalently

$$b_i^* = \sum_{j\in U_i^0} \big(\pi_j(z_j^{**}(U_i^0)) - \bar{\mathbf{c}} \cdot z_j^{**}(U_i^0)\big) - \sum_{j\in U_i} \big(\pi_j(z_j^{**}(U_i)) - \bar{\mathbf{c}} \cdot z_j^{**}(U_i)\big) > 0$$

[10] For $T \in \mathcal{P}_S$ such that $1 \in T$ there exists a unique $(z_i^*(S))_{i\in T}$ since the proof is the same as that of Proposition 1. For $T' \in \mathcal{P}_S$ such that $1 \notin T'$, there exists a unique $(z_i^*(S))_{i\in T'}$ since the proof is the same as that appearing in Ambec and Sprumont (2002, pp.456-457).

[11] The proof is the same as that of Proposition 1.

Proof. See Appendix B. ■

Since the constrained claim vector satisfies both the core lower bound for N and the aspiration upper bound for N, it is an efficient benefit. Furthermore, we remark on the proof. If extraction cost $\bar{\mathbf{c}}$ is zero and there is no assumption of essential water consumption, then the proof of Theorem 2 appearing in the present paper reduces to the proof of the theorem appearing in Ambec and Sprumont (2002).[12]

Finally, we demonstrate how to determine the share ratio α under the potential utilization of waters among the states. Let $n \geq 3$. Under the core lower bounds and the aspiration upper bounds, for each $w \in \mathcal{W}$, each $(c, E) \in \mathcal{P}$, and each $i \in N$, the outcome chosen by the *α-egalitarian proportional rule* is given by

$$\varphi_i^\alpha(b^*, E) = \alpha b_i^* + (1-\alpha)\frac{E}{n}.$$

As stated in the Introduction, all the states who are the members of the commission of an international river vote on water management based on the environmental law. Suppose that states are making decision about where to put a share ratio α on the interval $[0, 1]$. Each state i's bliss point is $\alpha = 1$ if $b_i^* > \frac{E}{n}$; $\alpha = 0$ if $b_i^* < \frac{E}{n}$; and $\alpha = [0, 1]$ if $b_i^* = \frac{E}{n}$. This observation means that preferences are *single-peaked* over a single-dimensional space. The two candidates of the outcome chosen by the α-egalitarian proportional rule are the contained claim vector and the equal division. Suppose that voting is the majority rule. Thanks to the Median Voter Theorem[13], we can determine the share ratio $\alpha = 1$ or $\alpha = 0$. That is, the constrained claim vector is chosen as the legal and political agreement of water problems if the states who prefer $\alpha = 1$ to $\alpha = 0$ consist of a majority. Otherwise, the equal award division is chosen as the legal and political agreement of water problems.

5 Concluding remarks on related literature

Next, it is worth comparing our study with several related papers except for Ambec and Sprumont (2002). Since the seminal paper by Ambec and Sprumont (2002), the axiomatic literature on water problems has been growing. Under the model where each state's benefit function exhibits a satiation point, Ambec and Ehlers (2008) characterize a welfare distribution that coincides

[12] In Ambec and Sprumont (2002), the theorem shows the unique existence of a downstream incremental distribution.

[13] For the details of the Median Voter Theorem, for instance, see Austen-Smith and Banks (2000).

with the downstream incremental distribution.[14] Using the assumptions of benefit functions appearing in Ambec and Ehlers, van den Brink, van der Laan and Moes (2012) characterize the set of certain welfare distributions including the downstream incremental distribution in the case of *multiple* watercourses. Under the assumptions of concavity and continuity of benefit functions, van den Brink, Estévez-Fernández, van der Laan, and Moes (2014) characterize certain fair allocation rules by independent axioms imposed on water problems. However, these papers do not give us any insight into either how to solve water shortage issues or axiomatizations of the α-egalitarian proportional rule for water problems. On the other hand, Ansink and Weikard (2012) characterize the class of sequential sharing rules, including the proportional rule, for claims problems for watercourse states. Ansink and Weikard (2012) assume that each state has a claim to its initial endowment, whereas we assume that each state has a claim to its benefit derived from water problems in the context of Ambec and Sprumont (2002).

Finally, we remark on an open question: Since we deal with only the case of a single watercourse, whether or not we can generalize the results of our paper to water problems with multiple watercourses may deserve investigation, which we leave to the future research. For this future research, an extension of our model by means of a game theoretic approach with a permission structure may be useful.[15]

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[14] For the definition of the downstream incremental distribution, see footnote 9 and Definition 1.

[15] For a game theoretic approach of river problems with a permission structure, see van den Brink, He, and Huang (2017).

Appendix A: The logical independence

For checking the logical independence of the four axioms, we consider the following four rules.

- For each $w \in \mathcal{W}$ such that $n \geq 3$, each $(c, E) \in \mathcal{P}$, and each $i \in N$, let $\varphi_i^1(c, E) = \frac{E}{n+1}$. The mapping φ^1 satisfies all the axioms except for *efficiency*.
- For each $w \in \mathcal{W}$ such that $n \geq 3$, and each $(c, E) \in \mathcal{P}$, let $\varphi^2(c, E) = b^*$, where b^* is a (unique) constrained claim vector defined in Section 4. The mapping φ^2 satisfies all the axioms except for *anonymity*.
- For each $w \in \mathcal{W}$ such that $n \geq 3$, and each $(c, E) \in \mathcal{P}$, let $\varphi^3(c, E) = EA(c, E)$ if $c = b^*$; otherwise $\varphi^3(c, E) = PR(c, E)$. Note that b^* is a (unique) constrained claim vector defined in Section 4. The mapping φ^3 satisfies all the axioms except for *continuity*. In fact, for $(c, E) \in \mathcal{P}$ such that $(c_1, \cdots, c_{i-1}, c_{i+1}, \cdots, c_n) = (b_1^*, \cdots, b_{i-1}^*, b_{i+1}^*, \cdots, b_n^*)$ and for $c_i \to b_i^*$, $\varphi^3(c, E) \to b^*$. On the other hand, for $(c, E) \in \mathcal{P}$ such that $c = b^*$, $\varphi^3(c, E) = EA(c, E)$. Thus, $\varphi^3(c, E)$ does not satisfy quasi-continuity, which implies that it does not satisfy *continuity*.
- For each $w \in \mathcal{W}$ such that $n \geq 3$, and each $(c, E) \in \mathcal{P}$, let $\varphi^4(c, E)$ be given by the **Talmud rule** (Aumann and Maschler 1985), denoted T, that is, for each $i \in N$ (1) if $\sum(c_i/2) \geq E$, then $T_i(c, E) \equiv \min\{c_i/2, \lambda\}$, where λ is chosen so that $\sum_N \min\{c_i/2, \lambda\} = E$; (2) if $\sum(c_i/2) \leq E$, then $T_i(c, E) \equiv c_i - \min\{c_i/2, \lambda\}$, where λ is chosen so that $\sum_N [c_i - \min\{c_i/2, \lambda\}] = E$. The mapping φ^4 satisfies all the axioms except for *reallocation-proofness*. In fact, for $(c, E) \in \mathcal{P}$ such that $c = (100, 200, 300)$ and $E = 200$, $T(c, E) = (50, 75, 75)$. On the other hand, for $(c, E) \in \mathcal{P}$ such that $c' = (150, 150, 300)$ and $E = 200$, $T(c', E) = (200/3, 200/3, 200/3)$. Therefore, $\sum_{i\in\{1,2\}} T(c, E) \neq \sum_{i\in\{1,2\}} T(c', E)$.

Appendix B: Proof of Theorem 2

For the proof, we have six claims.

Claim 1 If $(b_1, \cdots, b_n) \in \mathbb{R}^n$ satisfies the core lower bounds and the aspiration upper bounds, then for each $i \in N$ $b_i = b_i^*$.

Let $v(S) \equiv \sum_{i\in S} (\pi_i(z_i^*(S)) - \bar{\mathbf{c}} \cdot z_i^*(S))$, and $w(S) \equiv \sum_{i\in S} (\pi_i(z_i^{**}(S)) - \bar{\mathbf{c}} \cdot z_i^{**}(S))$. First, $v(\{1\}) = w(\{1\}) = b_1^*$. Therefore, $b_1 = b_1^*$. Next, fix j such that $j <$

n. Suppose that for each $i \leq j$ $b_i = b_i^*$. Since $v(U^0_{(j+1)}) = w(U^0_{(j+1)}) = \sum_{i\in U^0_{(j+1)}} b_i$, $b_{j+1} = v(U^0_{(j+1)}) - \sum_{i\in U^0_j} b_i$. By the supposition, $\sum_{i\in U^0_j} b_i = \sum_{i\in U^0_j} b_i^* = v(U_{(j+1)})$. Therefore, $b_{j+1} = v(U^0_{(j+1)}) - v(U_{(j+1)}) = b^*_{j+1}$.

Claim 2 v is "superadditive", that is, for each $S, T \subseteq N$ with $S \cap T = \emptyset$,

$$\begin{aligned}&\sum_{i\in S\cup T} (\pi_i(z_i^*(S\cup T)) - \bar{\mathbf{c}} \cdot z_i^*(S\cup T))\\ \geq\ &\sum_{i\in S} (\pi_i(z_i^*(S)) - \bar{\mathbf{c}} \cdot z_i^*(S)) + \sum_{i\in T} (\pi_i(z_i^*(T)) - \bar{\mathbf{c}} \cdot z_i^*(T)).\end{aligned}$$

Since $\sum_{i\in S\cup T} z_i^*(S\cup T) = \sum_{i\in S} z_i^*(S) + \sum_{i\in T} z_i^*(T) = \sum_{i\in S\cup T} e_i$, we show that

$$\sum_{i\in S\cup T} \pi_i(z_i^*(S\cup T)) \geq \sum_{i\in S} \pi_i(z_i^*(S)) + \sum_{i\in T} \pi_i(z_i^*(T)).$$

If $S\cup T$ is not consecutive, $\sum_{i\in S\cup T} \pi_i(z_i^*(S\cup T)) = \sum_{i\in S} \pi_i(z_i^*(S)) + \sum_{i\in T} \pi_i(z_i^*(T))$. If $S\cup T$ is consecutive and $1 \notin S\cup T$, by the the definition of z^*, $\sum_{i\in S\cup T} \pi_i(z_i^*(S\cup T)) \geq \sum_{i\in S} \pi_i(z_i^*(S)) + \sum_{i\in T} \pi_i(z_i^*(T))$. Without loss of generality, let $1 \in S$. It suffices to consider the case where S, T, and $S \cup T$ are consecutive. There is a pair of the lists of positive numbers $\{(\epsilon_i)_{i\in S}, (\epsilon'_i)_{i\in T}\}$ such that $\sum_{i\in S} \epsilon'_i = \sum_{i\in T}(\bar{x}_i + \epsilon_i - z_i^*(T))$ and for each $i \in S$ $z_i^*(S) - \epsilon'_i > \bar{x}_i$. This fact follows from the followings: Since for each $i \in T$ $\bar{x}_i > z_i^*(T)$, it suffices to show that $\sum_{i\in S}(z_i^*(S) - \bar{x}_i) > \sum_{i\in T}(\bar{x}_i - z_i^*(T))$. Suppose not, that is, for some S, T such that (i) S, T, and $S \cup T$ are consecutive, and (ii) $1 \in S$, $\sum_{i\in S}(z_i^*(S) - \bar{x}_i) \leq \sum_{i\in T}(\bar{x}_i - z_i^*(T))$, which implies that $\sum_{i\in S}(e_i - \bar{x}_i) \leq \sum_{i\in T}(\bar{x}_i - e_i)$. By this fact together with the assumption that $\sum_{i\in S\cup T} \bar{x}_i \leq \sum_{i\in S\cup T} e_i$, $\sum_{i\in S\cup T}(e_i - \bar{x}_i) = 0$. If so, we have that $\sum_{i\in S\cup T} z_i^*(S\cup T) = \sum_{i\in S\cup T} \bar{x}_i$, a contradiction to the assumption that $\sum_{i\in S\cup T} z_i^*(S \cup T) > \sum_{i\in S\cup T} \bar{x}_i$. Thus there is a pair of the lists of positive numbers $\{(\epsilon_i)_{i\in S}, (\epsilon'_i)_{i\in T}\}$ such that $\sum_{i\in S}(z_i^*(S) - \bar{x}_i) > \sum_{i\in T}(\bar{x}_i - z_i^*(T)) + \sum_{i\in T} \epsilon_i = \sum_{i\in S} \epsilon'_i$. For such a pair $\{(\epsilon_i)_{i\in S}, (\epsilon'_i)_{i\in T}\}$,

$$\begin{aligned}&\sum_{i\in S} \pi_i(z_i^*(S)) + \sum_{i\in T} \pi_i(z_i^*(T))\\ \leq\ &\sum_{i\in S} \pi_i(z_i^*(S) - \epsilon'_i) + \sum_{i\in T} \pi_i(\bar{x}_i + \epsilon_i)\\ \leq\ &\sum_{i\in S\cup T} \pi_i(z_i^*(S\cup T)) \text{ (by the definition of } z^*\text{)},\end{aligned}$$

which is a desired claim. Note that the first inequality is derived from the assumption that for each pair $\{i, j\}$ such that $i, j \in N$ and $i > j$, and for

$x_j > \bar{x}_j$, there is a positive ϵ such that $\pi'_i(\bar{x}_i + \epsilon) > \pi'_j(x_j)$ (See Section 2).

Claim 3 b^* satisfies the core lower bounds.

Since v is *superadditive* by Claim 2, it suffices to show that the core lower bounds hold for *consecutive* coalitions. Let $\min S$ and $\max S$ be the smallest member of S and the largest member of S, respectively. For any consecutive S such that $1 \notin S$, $\{S, U_{\min S}\}$ is a partition of $U^0_{\max S}$. By the definitions of b^* and v, $\sum_{i \in S} b^*_i = v(U^0_{\max S}) - v(U_{\min S})$. Since v is superadditive, $v(U^0_{\max S}) - v(U_{\min S}) \geq v(S)$, which implies that b^* satisfies the core lower bounds.

Claim 4 For $S \subset T \subset N$ and $i > \max T$, $w(S \cup \{i\}) - w(S) \geq w(T \cup \{i\}) - w(T)$.

The proof of Claim 4 consists of two steps:

Step 1 If $\emptyset \neq S \subset T \subset N$, then $z^{**}(S) \geq (z^{**}_k(T))_{k \in S}$.

It suffices to show that $z^{**}(S) \geq (z^{**}_k(T))_{k \in S}$ whenever $\emptyset \neq S \neq N$ and $t \in N \backslash S$. Write $z^{**}(S) = x$ and $(z^{**}_k(S \cup \{t\}))_{k \in S} = y$. We claim $\sum_{k \in S}(y_k - x_k) \leq 0$. Suppose $\sum_{k \in S}(y_k - x_k) > 0$. By the definition of w, $\sum_{k \in S} x_k = \sum_{k \in U^0_{\max S}} e_k$, which implies $\sum_{k \in S} y_k > \sum_{k \in U^0_{\max S}} e_k$, a contradiction to the constraint $\sum_{k \in S} y_k \leq \sum_{k \in U^0_{\max S}} e_k$. Let $k_1 \leq \cdots \leq k_L$ be those $k \in S$ such that $x_k \neq y_k$ (if none exists, there is nothing to prove). We claim $y_{k_L} - x_{k_L} < 0$. Suppose, by contradiction, $y_{k_L} - x_{k_L} \geq 0$ and $x_{k_L} \neq y_{k_L}$. Let j^* be the largest member in U_{k_L} such that $y_{j^*} - x_{j^*} < 0$. (Note that if $j^* = k_L$, $y_{k_L} - x_{k_L} < 0$, there is nothing to prove by using contradiction. If $j^* \neq k_L$, j^* necessarily exists since $\sum_{k \in S}(y_k - x_k) \leq 0$ and $y_{k_L} - x_{k_L} > 0$.) Let $y^\epsilon \in \mathbb{R}^S_+$ such that $y^\epsilon_{k_L} \equiv y_{k_L} - \epsilon$, $y^\epsilon_{j^*} \equiv y_{j^*} + \epsilon$, and $y^\epsilon_k \equiv y_k$ for $k \neq k_L, j^*$. Since $y_{j^*} < x_{j^*}$, $x_{k_L} < y_{k_L}$ and $\pi'_{j^*}(x_{j^*}) = \pi'_{k_L}(x_{k_L})$ (by the argument in the proof of Proposition 1), $\pi'_{j^*}(y_{j^*}) > \pi'_{j^*}(x_{j^*}) = \pi'_{k_L}(x_{k_L}) > \pi'_{k_L}(y_{k_L})$. Using this observation and the strict concavity of benefit functions, we can choose $\epsilon > 0$ small enough so that

$$
\begin{aligned}
&\sum_{k \in S}[(\pi_k(y^\epsilon_k) - \bar{\mathbf{c}} \cdot y^\epsilon_k) - (\pi_k(y_k) - \bar{\mathbf{c}} \cdot y_k)] \\
&= [\pi_{j^*}(y^\epsilon_{j^*}) - \bar{\mathbf{c}} \cdot y^\epsilon_{j^*} - (\pi_{j^*}(y_{j^*}) - \bar{\mathbf{c}} \cdot y_{j^*})] + [\pi_{k_L}(y^\epsilon_{k_L}) - \bar{\mathbf{c}} \cdot y^\epsilon_{k_L} - (\pi_{k_L}(y_{k_L}) - \bar{\mathbf{c}} \cdot y_{k_L})] \\
&= [\pi_{j^*}(y^\epsilon_{j^*}) - \pi_{j^*}(y_{j^*})] + [\pi_{k_L}(y^\epsilon_{k_L}) - \pi_{k_L}(y_{k_L})] - \bar{\mathbf{c}} \cdot (y^\epsilon_{j^*} - y_{j^*}) - \bar{\mathbf{c}} \cdot (y^\epsilon_{k_L} - y_{k_L}) \\
&= [\pi_{j^*}(y^\epsilon_{j^*}) - \pi_{j^*}(y_{j^*})] + [\pi_{k_L}(y^\epsilon_{k_L}) - \pi_{k_L}(y_{k_L})] \\
&> 0,
\end{aligned}
$$

while y^ϵ meets the same constraints as y. Note that the inequality is derived from $y^\epsilon_{j^*} > y_{j^*}$, $y^\epsilon_{k_L} < y_{k_L}$, $\pi'_{j^*}(y_{j^*}) > \pi'_{k_L}(y_{k_L})$, and strict concavity of benefit functions. Thus, we have a contradiction to the optimal solution y. Because $y_{k_L} - x_{k_L} < 0$, it follows that $y_{k_l} - x_{k_l} < 0$ successively for $l = L - 1, \cdots, 1$.

Step 2 For $S \subset T \subset N$ and $i > \max T$, $w(S\cup\{i\})-w(S) \geq w(T\cup\{i\})-w(T)$.

Let $S \subset T \subset N$, and $i > \max T$. Let $z' \in \mathbb{R}_+^{S\cup\{i\}}$ such as $z'_i = z_i^{**}(T \cup \{i\})$, and for each $j \in S$ $z'_j = z_j^{**}(T \cup \{i\}) + z_j^{**}(S) - z_j^{**}(T)$. By Step 1, for each $j \in S$ $z_j^{**}(T \cup \{i\}) \leq z_j^{**}(T) \leq z_j^{**}(S)$. Therefore, for each $j \in S$ $0 \leq z_j^{**}(T \cup \{i\}) \leq z'_j \leq z_j^{**}(S)$. Since for each $j \in S$ $z'_j \leq z_j^{**}(S)$, state j's consumption plan z'_j for $S \cup \{i\}$ satisfies the same constraints as $z_j^{**}(S)$. By the definition of z^{**}, for each $j \in S$ $z_j^{**}(S)$ satisfies the same constraints as $z_j^{**}(S \cup \{i\})$. Again by the definition of z^{**}, $z_i^{**}(S) \cup \{i\}$ satisfies the same constraints as $z_i^{**}(T \cup \{i\})$. Therefore, the consumption plan z' for $S \cup \{i\}$ satisfies the same constraints as $z^{**}(S \cup \{i\})$, namely, for each $l \in S \cup \{i\}$ $\sum_{k\in U_l^0 \cap (S\cup\{i\})} z'_k \leq \sum_{k\in U_l^0} e_k$. By this observation together with the definition of w, $w(S \cup \{i\}) \geq \sum_{j\in S\cup\{i\}} \left(\pi_j(z'_j) - \bar{\mathbf{c}} \cdot z'_j\right)$, which implies that

$$w(S \cup \{i\}) - w(S) \geq \sum_{l\in S\cup\{i\}} (\pi_l(z'_l) - \bar{\mathbf{c}} \cdot z'_l) - \sum_{j\in S} \left(\pi_j(z_j^{**}(S)) - \bar{\mathbf{c}} \cdot z_j^{**}(S)\right)$$
$$= \pi_i(z'_i) - \bar{\mathbf{c}} \cdot z'_i + \sum_{j\in S} \left[\left(\pi_j(z'_j) - \pi_i(z_j^{**}(S))\right) - \bar{\mathbf{c}} \cdot \left(z'_j - z_j^{**}(S)\right)\right].$$

On the other hand,

$$\begin{aligned}
&w(T \cup \{i\}) - w(T)\\
&= \sum_{l\in S\cup\{i\}} (\pi_l(z_l^{**}(T \cup \{i\})) - \bar{\mathbf{c}} \cdot z_l^{**}(T \cup \{i\})) + \sum_{l\in T\setminus S} (\pi_l(z_l^{**}(T \cup \{i\})) - \bar{\mathbf{c}} \cdot z_l^{**}(T \cup \{i\}))\\
&- \sum_{j\in S} \left(\pi_j(z_j^{**}(T)) - \bar{\mathbf{c}} \cdot z_j^{**}(T)\right) - \sum_{l\in T\setminus S} (\pi_l(z_l^{**}(T)) - \bar{\mathbf{c}} \cdot z_l^{**}(T))\\
&= \pi_i(z'_i) - \bar{\mathbf{c}} \cdot z'_i + \sum_{j\in S} \left[\left(\pi_j(z_j^{**}(T \cup \{i\})) - \pi_j(z_j^{**}(T))\right) - \bar{\mathbf{c}} \cdot \left(z_j^{**}(T \cup \{i\}) - z_j^{**}(T)\right)\right]\\
&+ \sum_{l\in T\setminus S} [(\pi_l(z_l^{**}(T \cup \{i\})) - \pi_l(z_l^{**}(T))) - \bar{\mathbf{c}} \cdot (z_l^{**}(T \cup \{i\}) - z_l^{**}(T))]\\
&\leq \pi_i(z'_i) - \bar{\mathbf{c}} \cdot z'_i + \sum_{j\in S} \left[\left(\pi_j(z_j^{**}(T \cup \{i\})) - \pi_j(z_j^{**}(T))\right) - \bar{\mathbf{c}} \cdot \left(z_j^{**}(T \cup \{i\}) - z_j^{**}(T)\right)\right],
\end{aligned}$$

where the inequality follows from the fact that for each $l \in T\setminus S$,

$$(\pi_l(z_l^{**}(T)) - \pi_l(z_l^{**}(T \cup \{i\}))) - \bar{\mathbf{c}} \cdot (z_l^{**}(T) - z_l^{**}(T \cup \{i\})) \geq 0,$$

since $z_l^{**}(T \cup \{i\}) \leq z_l^{**}(T)$ (by Step 1) and $\pi'_l(z_l^{**}(T)) \geq \bar{\mathbf{c}}$ (by the same argument as in the proof of Proposition 1), and benefit function π_l is strictly concave. Since for each $j \in S$ benefit function π_j is strictly concave, $z_j^{**}(T \cup \{i\}) \leq z'_j \leq z_j^{**}(S)$, $z'_j - z_j^{**}(S) = z_j^{**}(T \cup \{i\}) - z_j^{**}(T)$ (by the definition of

z'), and $\pi'_j(z_j^{**}(T \cup \{i\})) \geq \pi'_j(z'_j) \geq \bar{\mathbf{c}}$ (by the fact that $\pi'_j(z_j^{**}(S)) \geq \bar{\mathbf{c}}$, and continuity and strict concavity of π_j),

$$\begin{aligned}
&\left(\pi_j(z'_j) - \pi_i(z_j^{**}(S))\right) - \bar{\mathbf{c}} \cdot \left(z'_j - z_j^{**}(S)\right) \\
&\geq \left(\pi_j(z_j^{**}(T \cup \{i\})) - \pi_i(z_j^{**}(T))\right) - \bar{\mathbf{c}} \cdot \left(z_j^{**}(T \cup \{i\}) - z_j^{**}(T)\right).
\end{aligned}$$

Therefore,

$$\begin{aligned}
& w(S \cup \{i\}) - w(S) \\
\geq\ & \pi_i(z'_i) - \bar{\mathbf{c}} \cdot z'_i + \sum_{j \in S} \left[\left(\pi_j(z'_j) - \pi_i(z_j^{**}(S))\right) - \bar{\mathbf{c}} \cdot \left(z'_j - z_j^{**}(S)\right)\right] \\
\geq\ & \pi_i(z'_i) - \bar{\mathbf{c}} \cdot z'_i + \sum_{j \in S} \left[\left(\pi_j(z_j^{**}(T \cup \{i\})) - \pi_i(z_j^{**}(T))\right) - \bar{\mathbf{c}} \cdot \left(z_j^{**}(T \cup \{i\}) - z_j^{**}(T)\right)\right] \\
\geq\ & w(T \cup \{i\}) - w(T),
\end{aligned}$$

which completes the proof of the claim.

Claim 5 b^* satisfies the aspiration upper bounds.

By the definition of b^* and Claim 4, for each $S \subseteq N$,

$$\sum_{i \in S} b_i^* = \sum_{i \in S} [w(U_i^0) - w(U_i)] \leq \sum_{i \in S} [w(U_i^0 \cap S) - w(U_i \cap S)] = w(S),$$

where the inequality is derived from the fact that for each $i \in S$ $(U_i \cap S) \subset S$ together with Claim 4, and the last equality is derived from the fact that for each $i \in S$ $U_i \cap S = U^0_{(\max(U_i \cap S))} \cap S$, so that all terms except for $w(U^0_{(\max S)} \cap S)$ and $w(U_{(\min S)} \cap S)$ cancel out, and $w(U^0_{(\max S)} \cap S) = w(S)$ and $w(U_{(\min S)} \cap S) = w(\emptyset) = 0$. Therefore, z^* satisfies the aspiration upper bounds.

Claim 6 For each $i \in N$, $b_i^* > 0$.

By the definition of b_1^*, $b_1^* > 0$. Again, by the definition of b_i^* and the superadditivity of v, for each $i \in N \backslash \{1\}$,

$$b_i^* = v(U_i^0) - v(U_i) \geq v(\{i\}) > 0,$$

where the last inequality is derived from the fact that $z^*(\{i\}) > 0$. ■

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